

A Rotating Quantum Vacuum

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Abstract

We investigate how a uniformly rotating frame is defined as the rest frame of an observer rotating with constant angular velocity Ω around the z axis of an inertial frame. Assuming that this frame is a Lorentz one, we second quantize a free massless scalar field in this rotating frame and obtain that creation-annihilation operators of the field are not the same as those of an inertial frame. This leads to a new vacuum state — a rotating vacuum — which is a superposition of positive and negative frequency Minkowski particles. After this, introducing an apparatus device coupled linearly with the field we obtain that there is a strong correlation between number of rotating particles (in a given state) obtained via canonical quantization and via response function of the rotating detector. Finally, we analyse polarization effects in circular accelerators in the proper frame of the electron making a connection with the inertial frame point of view.

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1 Introduction

1.1 Introductory Remarks

The purpose of this paper is to discuss the puzzle of the rotating detector [1] and to relate this to polarization effects of electrons in storage rings [2]. We try to avoid many technical difficulties to emphasize only fundamental results.

The most important step in the development of general relativity from special relativity is to accept the idea that is possible to discuss physics — and compare measurements — not only between inertial frames but also between any arbitrary frames of references. It is natural to ask how to second quantize any field in an arbitrary frame in Minkowski spacetime. Note that the principle of general covariance require that physically observables are always expressible in coordinate independent fashion. The development of such ideas introduce a plethora of new phenomena. One of these is the Unruh-Davies effect [3, 4]. An universal definition of the vacuum for a system described by a Hamiltonian is that the vacuum is the lowest energy state. If to describe the system we use a finite number of degrees of freedom, all representations of the operator's algebra are unitarily equivalents, i.e., different vacua lie in the same Hilbert space. This means that the physical description of the system will not depend on the choice of representation. However, if to describe the system we have to make use of infinite degrees of freedom, there are an infinite number of unitary inequivalent representations of the commutation relations [5]. Different inequivalent representations will in general give rise to different pictures with different physical implications.

A well known example of this situation arises in the study of the quantization of a field by observers with linear proper acceleration [6]. If we quantize a field in the Rindler's frame one finds quantization structure identical to the quantization obtained by inertial observers. As in the case of inertial observers, in the Rindler's manifold there is a time-like Killing vector and the symmetry generated by this vector field is implemented by a unitary operator group. The generator of this unitary group is positive definite and the construction of eigenstates of this operator allows a particle interpretation where a new vacuum state (the Fulling vacuum) is introduced [7]. The Minkowski and the Fulling vacua are non-unitarily equivalents. It is possible to show that the Minkowski vacuum can be expressed into a set of EPR type of Rindler's particles [8]. As a natural consequence of this fact is that a particle detector at rest in Rindler's spacetime interacting with a massless scalar field prepared in the Minkowski vacuum responds as though is were at rest in Minkowski spacetime immersed in a bath of thermal radiation. Many authors claim that this case of linear acceleration is physically not very interesting since we need an eternal phase of constant acceleration.

A more tractable case (at least experimentally) is the case of transverse acceleration found in circular movement. This particular situation introduce some interesting questions related with the meaning of particles in non-inertial frames of references. It has been suggested that the answer to the question: how would a particle detector responds in a given situation? can elucidate this problem. As we will see, this question introduce the rotating detector puzzle. To understand the problem of the rotating detector we have to go back to the problem of the rotating disc, i.e., the problem of rotation in relativity. A

question that has interested many authors is whether the intrinsic geometry of a rotating disc is Euclidean or not. Infeld, using Einstein arguments [9] sustained that a rigid disc under uniform angular rotation Ω relative to an inertial frame will exhibit a non-Euclidean geometry (by a rigid body one understood a body in which during the motion no elastic stresses arises). The argument is that the circumference will suffer a Lorentz contraction although the radius r will not. Consequently, the circumference of the rotating disc relative to an inertial frame is less than $2\pi r$. Lorentz had a opposed point of view [10] and claimed that the intrinsic geometry of the rotating disc is Euclidean since the radius and the circumference of the rotating disc contract by the same amount.

A different approach to study this problem based on kinematic arguments has been presented by Hill long time ago [12]. If the speed of any point in an uniform rotating disc is a *linear* function of the radius, distant points have speeds exceeding the velocity of light. Hence this author concluded that the speed-distance law must be non-linear and approach the velocity of light when the radius goes to infinity. Even today these are open questions and no definite answer has been given.

The key point of the discussion is to find the correct transformation from an inertial frame to a local frame in any point of the disk with constant angular velocity Ω . Let $\Sigma^0 = (t', r', \theta', z')$ be an inertial frame with cylindrical coordinates r', θ', z' and the common frame time t' measured by synchronised standard clocks. Consider a point P revolving around the z' axis at fixed distance r'_P with constant angular velocity Ω_P with respect to Σ^0 , i.e.,

$$\frac{d\theta'_P}{dt'} = \Omega_P \quad (1)$$

$$\frac{dr'_P}{dt'} = \frac{dz'_P}{dt'} = 0. \quad (2)$$

In the rest frame of P , Σ_P we introduce the coordinates $t^{(r')}$, r , $\theta^{(r')}$ and z with $z' = z$ and $r' = r$. Note that $t^{(r')}$ is the local time variable at any point $r = r'$ and $\theta^{(r')}$ is the local angular variable at any point $r = r'$. If someone assume that the rotating frame is a Galilean one and quantize a massless scalar field in such frame, it is possible to show that the rotating vacuum is just the Minkowski vacuum. The mistake lies in the fact of use a Galilean tranformation for the introduction of coordinates in the rotating frame. The price we have to pay is that an aparatus device (a detector coupled with a field) which gives information about the particle content of the state of the field can be excited if it is prepared in the ground state with the field in the Minkowski vacuum [13]. This is an odd result. One would expect the rotating detector *not to be excited* by the rotating vacuum. In this paper we will try to sheed some light on these problems.

We would like to stress that we are not interested here in discussing the subtle problem of how to decode the information stored in the composite system (detector and scalar field) to convert in a classical sign. The modern treatment of this problem is the following: both the detector and the scalar field are not closed systems but they are open systems interacting with the enviroment. In this way certain phase relations disappear, i.e., loss of coherence to its enviroment (Decoherence). This idea allows that the composite system (detector and the scalar field) be described by a diagonal matrix density [14]. For an application of such ideas in the Unruh-Davies effect see for example Ref. [15].

1.2 Synopsis

The paper is organized as follows. In section 2 we discuss how a uniformly rotating frame is defined as the rest frame of an observer rotating with constant angular velocity Ω around the z' axes of an inertial frame Σ^0 . We first assume that the rotating frame is a Galilean one and second quantize a massless scalar field in such frame. We show that the rotating vacuum is just the Minkowski vacuum. Some disturbing situations are analysed. In section 3 we discuss radiative processes assuming that a uniformly rotating frame must be a Lorentz frame, i.e., we have to use a Lorentz-like transformation for the introduction of coordinates in the rotating frame. It is shown that the resulting rotating vacuum of the canonical quantization framework is not the Minkowski vacuum. In section 4 we perform the second quantization of the total Hamiltonian of the system to show that the process of an absorption (emission) of a rotating particle and excitation (decay) of the detector in the non inertial frame is interpreted as an excitation (decay) of the detector with emission of a Minkowski particle in the inertial frame. Conclusions are given in section 5. In this paper we use $\hbar = c = 1$.

2 The Rotating Frame Candidates

The problem of the rotating disc have been investigated by many authors and can be posed in the following way. Suppose the Minkowski spacetime with line element in the cylindrical coordinate system $x'^\mu = (t', r', \theta', z')$ adapted to an inertial frame given by

$$ds^2 = dt'^2 - dr'^2 - r'^2 d\theta'^2 - dz'^2. \quad (3)$$

Suppose a disc rotating uniformly about the z axis with angular velocity Ω . How a uniformly rotating frame is defined, i.e., the rest frame of an observer rotating with constant angular velocity Ω around the z' axes of an inertial frame Σ^0 ? In order words, we have to find the correct transformations formula for the transition $\Sigma^0 \rightarrow \Sigma^r$, where Σ^r is the local frame at any point of the disk with angular velocity Ω . Which mapping we have to assume to compare measurements made in an inertial and in a rotating frame of reference? Eddington [16], Rosen [17] and Landau and Lifshitz [18] adopted a coordinate system adapted to the rotating disk in such that transformation law between the cylindrical coordinate system $x'^\mu = (t', r', \theta', z')$ adapted to an inertial frame and rotating coordinate system $x^\mu = (t, r, \theta, z)$ adapted to the disk are given by:

$$t = t', \quad (4)$$

$$r = r', \quad (5)$$

$$\theta = \theta' - \Omega t', \quad (6)$$

$$z = z'. \quad (7)$$

In the rotating coordinate system $x^\mu = (t, r, \theta, z)$ the line element can be written as

$$ds^2 = (1 - \Omega^2 r^2) dt^2 - dr^2 - r^2 d\theta^2 - dz^2 + 2\Omega r^2 d\theta dt. \quad (8)$$

The line element in the rotating frame is stationary but not static. The world line of a point of the disc is an integral curve of the Killing vector $\xi = (1 - \Omega^2 r^2)^{-1/2} \partial/\partial t$ which is

timelike only for $\Omega r < 1$. Rosen claimed that using the transformations given by eqs.(4-7) the speed-distance law is linear and this put a limit on the size of the disc that rotate with a given angular velocity. The same point of view was given by Landau and Lifshitz.

A “more natural” way to investigate such problem is to looking for a Lorentz frame such that the velocity of the disk does not obey a linear velocity law. In this case we have to find a naturally adapted coordinate system to this infinite rotating disk without the problems found in the Landau’s *et al* coordinate system.

A second possibility trying to avoid the disc problem in the core of the discussion is to follow Hill’s arguments. This author presented a different answer for the problem. He raised the question if it is possible to find a group of transformation between the inertial and the non-inertial frame in such a way that for small velocities we obtain the linear speed-distance law and for large distance approach the speed of light. Such a transformation frames was presented by Trocherries [19] and also Takeno [20]. In Takeno’s derivation three assumptions are used: the transformation law constitute a group, for small velocities we recovered the usual linear velocity law ($v = \Omega r$) and velocity composition law is also in agreement with special relativity. One hopes that the transformation law derived assuming this rules is unique, although the proof of such assumption is missing.

The coordinate transformations derived by Takeno are given by

$$t = t' \cosh \Omega r' - r' \theta' \sinh \Omega r', \quad (9)$$

$$r = r', \quad (10)$$

$$\theta = \theta' \cosh \Omega r' - \frac{t'}{r'} \sinh \Omega r', \quad (11)$$

$$z = z'. \quad (12)$$

Note that if we assume this mapping to connect measurements made in the rotating frame and those made in the inertial frame, in the rotating coordinate system the line element assume a non-stationary form

$$ds^2 = dt^2 - (1 + P)dr^2 - r^2 d\theta^2 - dz^2 + 2Qdrd\theta + 2Sdrdt, \quad (13)$$

where P , Q and S are given by

$$P = \left(\frac{Y}{r^2} + 4\Omega\theta t\right) \sinh^2 \Omega r - \frac{\Omega}{r}(t^2 + r^2\theta) \sinh^2 2\Omega r + \Omega^2 Y, \quad (14)$$

$$Q = r\theta \sinh^2 \Omega r - \frac{1}{2}t \sinh 2\Omega r + \Omega r t, \quad (15)$$

$$S = \frac{t}{r} \sinh^2 \Omega r - \frac{1}{2}\theta \sinh 2\Omega r - \Omega r \theta, \quad (16)$$

and

$$Y = (t^2 - r^2\theta^2). \quad (17)$$

We would like to remark that this above line element define the intrinsic geometry of the rotating disk. Furthermore this line element define a multiple connect manifold. We will discuss this point further.

Before starting to analyse the detector problem we would like to present some experimental and theoretical arguments against and in favour of Trocherries and Takeno’s

coordinate transformation. The Special Theory of Relativity show us that different inertial frames are connected by Lorentz transformations. Why we use a Lorentz-like transformation to connect measurements in the inertial and the non-inertial frame? We should mention that it is possible to write the transformations defined by eqs.(9-12) making an analogy with the Lorentz transformations. Let us define $l = r\theta$ and $\gamma = (1 - v^2)^{-1/2}$. It is straightforward to show that eq.(9) and eq.(11) becomes

$$t = \gamma(t' - vl') \quad (18)$$

and

$$l = \gamma(l' - vt'). \quad (19)$$

In other words the transformations defined by Trocheris and Takeno are “Lorentz-like” transformation. The fundamental difference is that in this case the velocity is $v = \tanh \Omega r'$. It has been suggested by Phipps [21] that the Takeno’s velocity distribution does not agree with the experimental data. Strauss [11] also adopted a Lorentz-like transformation, but with a linear $v = \Omega r$ speed-distance law. The important consequence is that the light velocity on the rotating frame is one. Again, some authors claim that this result is in contradiction with the Sagnac’s effect [22, 23]. The only way to have results consistent with this effect is to use a “Galilean” transformation given by eqs.(4-7). We would like to stress that the above arguments does not establish conclusively that we have to use the Galilean transformations. As we will show, direct supports of Lorentz-like transformation between both frames are supplied by the rotating detector puzzle and the depolarization effect of electrons in a circular accelerator.

To investigate the meaning of particle in an arbitrary frame in a flat spacetime we have two different routs. The first is to canonical quantize the field and obtain the number of particles operator for each mode $N_R(\omega) = b^\dagger(\omega)b(\omega)$ in the arbitrary frame. For static line elements (Rindler, for example) this can be done in a unambiguously way. For time dependent line elements (Milne, for example) it is possible to define instantaneous positive and negative frequency modes and diagonalize an instantaneous Hamiltonian operator. The second rout is to introduce an measuring device, i.e., a detector (atoms) with a coupling with the field. Experimentalists detect photons in laboratories. They are absorbed at fixed instants and cause the electrons in the atoms to jump from a ground state to an excited state. Glauber and others produced a theory of photo-detection using the rotating-wave approximation (RWA). In this approximation the detector (square-law detector) must gives information about the particle content of some state [24, 25]. In other words, square-law detectors goes to excited state by absorption of quanta of the field.

Before continue let us discuss some arguments pro and contrary of the Glauber’s detector. Bykov and Takarskii [26] showed that this detector model violates the causality principle for short observations times. If we assume that the observation time is large compared with E^{-1} , everething is in order. Note that it is possible to consider measurements of finite durations only for $\Delta T > 1/E$. Of short time intervals we cannot even define the two-level system. Nevertheless there are some situations were we can not use the RWA, for example in resonant interaction between two atoms [27]. As we will see the RWA can not be used only to find the rate of spontaneous decay. The same situation

occur in a semi-classical theory of spontaneous emission where an atom in the excited state is stable since there is no vacuum fluctuations.

Going back to our problem, let us discuss these two routes that are usually used to investigate the meaning of particles in an arbitrary frame of reference. Let us first perform the quantization of a massless real scalar field in the Landau's rotating frame. First we have to solve the Klein-Gordon equation in the $x^\mu = (t, r, \theta, z)$ coordinate system given by

$$\left[\left(\frac{\partial}{\partial t} - \Omega \frac{\partial}{\partial \theta} \right)^2 - \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} - \frac{\partial^2}{\partial z^2} \right] \varphi(t, r, \theta, z) = 0 \quad (20)$$

to find the normal modes that satisfies

$$L_{\bar{K}} u_{\bar{q}mk_z}(t, r, \theta, z) = -i\bar{\omega} u_{\bar{q}mk_z}(t, r, \theta, z), \quad (21)$$

where \bar{K} is a time-like Killing vector. It is not difficult to show that the modes are given by [13, 28]

$$u_{\bar{q}mk_z}(t, r, \theta, z) = \frac{1}{2\pi [2(\bar{\omega} + m\Omega)]^{\frac{1}{2}}} e^{-i\bar{\omega}t} e^{im\theta} e^{ik_z z} J_m(qr) \quad (22)$$

where

$$(\bar{\omega} + m\Omega)^2 = k_z^2 + q^2, \quad (23)$$

$$(\bar{\omega} + m\Omega) > 0, \quad (24)$$

and the radial part $J_m(qr)$ are the Bessel functions of first kind [29]. To continue the canonical quantization, the field operator $\varphi(t, r, \theta, z)$ have to be expanded using these modes and the complex conjugates $\{u_{\bar{q}mk_z}(t, r, \theta, z), u_{\bar{q}mk_z}^*(t, r, \theta, z)\}$, i.e.,

$$\varphi(t, r, \theta, z) = \sum_m \int \bar{q} d\bar{q} dk_z \left[a_{\bar{q}mk_z} u_{\bar{q}mk_z}(t, r, \theta, z) + a_{\bar{q}mk_z}^\dagger u_{\bar{q}mk_z}^*(t, r, \theta, z) \right]. \quad (25)$$

Of course, in stationary geometries the definition of positive and negative frequency modes has no ambiguities. To compare both quantizations i.e., in the inertial and in the rotating frame, we have to find the Bogoliubov coefficients between the inertial modes (cylindrical waves) $\{\psi_k(t', r', \theta', z'), \psi_k^*(t', r', \theta', z')\}$ and the non-inertial ones given by eq.(22). Since the Bogoliubov coefficients $\beta_{k\nu} = -(u_{\bar{q}mk_z}, \psi_k)$ are zero, Letaw and Pfautsch concluded that the vacuum defined by

$$a(\bar{q}, m, k_z)|0, R\rangle = 0 \quad \forall \quad \bar{q}, m, k_z, \quad (26)$$

i.e., the rotating vacuum is just the Minkowski vacuum. Note that we are not interested to discuss complications introduced by infinite-volume divergences. To circumvent this problem the creation and annihilation operators have to be smeared with square integrable test functions (wave-packet).

The introduction of the detector in this quantization scheme raised a fundamental question. If we prepare a detector in the ground state and the field in the Minkowski vacuum there is a non-null probability to find the detector in the excited state if the detector travel in a rotating world line, parametrized by eqs.(4-7). The orbiting detector will "measure" quanta of the field although there is no rotating quanta in the Minkowski

vacuum. How is possible to a detector being excited if is traveling in a rotating disc if we prepare the field in the Minkowski vacuum? After the absorption, the field will be in a lower energy level than the “original vacuum”. Therefore this “original vacuum” is not the true vacuum of the field. Another way to formulate the problem is the following one: our physical intuition say that a a rotating particle detector in the ground state interacting with the scalar field prepared in the rotating vacuum must stay in the ground state. Nevertheless, assuming the Galilean frame, the Minkowski vacuum $|0, M\rangle$ is exactly the rotating vacuum $|0, R\rangle$ and the rate of excitations instead to be zero is different from zero. The detector behaves as if it is not coupled to the vacuum, concluded Davies, Dray and Manogue [1]. This is the so called rotating detector puzzle. Some time ago Grove and Ottewill trying to sheed some light for these problem studied extended detectors [30]. Letaw and Pfautsch, Padmanabhan [31] and also Padmanabhan and Singh [32] concluded that the correlation between vacuum states defined via canonical quantization and via detector is broken in this particular situation. We cannot agree with this conclusion. The preceding considerations suggest that we can not accept a Galilean rotating frame with a maximum radius given by $(R_{\max} = 1/\Omega)$. In the next section we will remember the formalism and discuss some possibilities to solve the rotating detector puzzle and the interpretational difficulties associated with it.

3 Radiative Processes of the Monopole Detector and a New Rotating Vacuum

Let us consider a system (a detector) endowed with internal degrees of freedom defining two energy levels with energy ω_g and ω_e , ($\omega_g < \omega_e$) and respective eigenstates $|g\rangle$ and $|e\rangle$ [4, 33, 34]. This system is weakly coupled with a hermitian massless scalar field $\varphi(x)$ with interaction Lagrangian

$$L_{int} = \lambda m(\tau) \varphi(x(\tau)), \quad (27)$$

where $x^\mu(\tau)$ is the world line of the detector parametrized using the proper time τ , $m(\tau)$ is the monopole operator of the detector and λ is a small coupling constant between the detector and the scalar field. For different couplings between the detector and the scalar field see for example Ref.[35] and also Ford and Roman [36].

In order to discuss radiative processes of the whole system (detector plus the scalar field), let us define the Hilbert space of the system as the direct product of the Hilbert space of the field \mathbf{H}_F and the Hilbert space of the detector \mathbf{H}_D

$$\mathbf{H} = \mathbf{H}_D \otimes \mathbf{H}_F. \quad (28)$$

The Hamiltonian of the system can be written as:

$$H = H_D + H_F + H_{int}, \quad (29)$$

where the unperturbed Hamiltonian of the system is composed of the noninteracting detector Hamiltonian H_D and the free massless scalar field Hamiltonian H_F . We shall define the initial state of the system as:

$$|\mathcal{T}_i\rangle = |j\rangle \otimes |\Phi_i\rangle, \quad (30)$$

where $|j\rangle$, ($j = 1, 2$) are the two possible states of the detector ($|1\rangle = |g\rangle$ and $|2\rangle = |e\rangle$) and $|\Phi_i\rangle$ is the initial state of the field. In the interaction picture, the evolution of the combined system is governed by the Schrodinger equation

$$i\frac{\partial}{\partial\tau}|\mathcal{T}\rangle = H_{int}|\mathcal{T}\rangle, \quad (31)$$

where

$$|\mathcal{T}\rangle = U(\tau, \tau_i)|\mathcal{T}_i\rangle, \quad (32)$$

and the evolution operator $U(\tau, \tau_i)$ obeys

$$U(\tau_f, \tau_i) = 1 - i \int_{\tau_i}^{\tau_f} H_{int}(\tau') U(\tau', \tau_i) d\tau'. \quad (33)$$

In the weak coupling regime, the evolution operator can be expanded in power series of the interaction Hamiltonian. To first order, it is given by

$$U(\tau_f, \tau_i) = 1 - i \int_{\tau_i}^{\tau_f} d\tau' H_{int}(\tau'). \quad (34)$$

The probability amplitude of the transition from the initial state $|\mathcal{T}_i\rangle = |j\rangle \otimes |\Phi_i\rangle$ at the hypersurface $\tau = 0$ to $|j'\rangle \otimes |\Phi_i\rangle$ at τ is given by

$$\langle j' \Phi_f | U(\tau, 0) | j \Phi_i \rangle = -i\lambda \int_0^\tau d\tau' \langle j' \Phi_f | m(\tau') \varphi(x(\tau')) | j \Phi_i \rangle, \quad (35)$$

where $|\Phi_f\rangle$ is an arbitrary state of the field and $|j'\rangle$ is the final state of the detector. The probability of the detector being excited at the hypersurface τ , assuming that the detector was prepared in the ground state is:

$$P_{eg}(\tau) = \lambda^2 |\langle e | m(0) | g \rangle|^2 \int_0^\tau d\tau' \int_0^\tau d\tau'' e^{iE(\tau'' - \tau')} \langle \Phi_i | \varphi(x(\tau')) \varphi(x(\tau'')) | \Phi_i \rangle, \quad (36)$$

where $E = \omega_e - \omega_g$ is the energy gap between the eigenstates of the detector. Note that we are interested in the final state of the detector and not that of the field, so we sum over all the possible final states of the field $|\Phi_f\rangle$. Since the states are complete, we have

$$\sum_f |\Phi_f\rangle \langle \Phi_f| = 1. \quad (37)$$

Eq.(36) shows us that the probability of excitation is determined by an integral transform of the positive Wightman function.

Before starting to analyze radiative processes, we would like to point out that a more realistic model of particle detector must also have a continuum of states. This assumption allows us to use a first order perturbation theory without taking into account higher order corrections. Although we will use in this paper the two-state model, the case of a mixing between a discrete and a continuum eigenstates deserves further investigations. For a complete discussion of the detector problem see for example Refs. [37, 38, 39]. In this section we will use a different notation. Two distincts spacetime points in the rotating

coordinate system will be given by $x^\mu = (\eta, \xi)$ and $x'^\mu = (\eta', \xi')$. We are using the variable ξ to represent both coordinates r and θ i.e. $\xi \equiv \{r, \theta\}$. In the applications to storage ring we will be not interested in the r and θ dependence of the probability transition and will use only $P_{12}(E, \Delta T)$. Since we are interested in finite time measurements let us follow Svaiter and Svaiter [40], and also Ford, Svaiter and Lyra [41] defining

$$\eta - \eta' = \zeta \quad (38)$$

and

$$\eta_f - \eta_i = \Delta T. \quad (39)$$

We would like to stress that Levin, Peleg and Peres [42] also used the same technique to study radiative processes in finite observation times. Substituting eq.(38) and eq.(39) in eq.(36) and defining $F(E, \Delta T, \xi, \xi')$ by

$$P_{12}(E, \Delta T, \xi, \xi') = \lambda^2 |\langle 2 | m | 1 \rangle|^2 F(E, \Delta T, \xi, \xi') \quad (40)$$

we have

$$F(E, \Delta T, \xi, \xi') = \int_{-\Delta T}^{\Delta T} d\zeta (\Delta T - |\tau|) e^{iE\zeta} \langle 0, M | \varphi(\eta', \xi') \varphi(\eta, \xi) | 0, M \rangle. \quad (41)$$

It is clear that $F(E, \Delta T, \xi, \xi')$ is the probability of excitation normalized by the selectivity of the detector. The same can be done for decay processes and the probability of decay $P_{21}(E, \Delta T, \xi, \xi')$ is given by

$$P_{21}(E, \Delta T, \xi, \xi') = \lambda^2 |\langle 1 | m | 2 \rangle|^2 F(E, \Delta T, \xi, \xi'). \quad (42)$$

Let us define the rate $R(E, \Delta T, \xi, \xi')$, i.e., this probability transition per unit proper time as:

$$R(E, \Delta T, \xi, \xi') = \frac{d}{d(\Delta T)} F(E, \Delta T, \xi, \xi'). \quad (43)$$

Writting in a concise form we have:

$$R(E, \Delta T, \xi, \xi') = \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \langle 0, M | \varphi(\eta', \xi') \varphi(\eta, \xi) | 0, M \rangle. \quad (44)$$

This important result shows that asymptotically the rate of excitation (decay) of the detector is given by the Fourier transform of the positive frequency Wightman function. This is exactly the quantum version of the Wiener-Khintchine theorem which asserts that the spectral density of a stationary random variable is the Fourier transform of the two point-correlation function. Spliting the field operator in positive and negative frequency parts, the rate becomes:

$$\begin{aligned} R(E, \Delta T, \xi, \xi') = & \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \left[\langle 0, M | \varphi^{(+)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) | 0, M \rangle \right. \\ & + \langle 0, M | \varphi^{(-)}(\eta', \xi') \varphi^{(-)}(\eta, \xi) | 0, M \rangle \\ & + \langle 0, M | \varphi^{(-)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) | 0, M \rangle \\ & \left. + \langle 0, M | \varphi^{(+)}(\eta', \xi') \varphi^{(-)}(\eta, \xi) | 0, M \rangle \right]. \end{aligned} \quad (45)$$

The last matrix element can be written as

$$\begin{aligned} \langle 0, M | \varphi^{(+)}(\eta', \xi') \varphi^{(-)}(\eta, \xi) | 0, M \rangle &= \langle 0, M | \varphi^{(-)}(\eta, \xi) \varphi^{(+)}(\eta', \xi') | 0, M \rangle \\ &+ [\varphi^{(+)}(\eta', \xi'), \varphi^{(-)}(\eta, \xi)]. \end{aligned} \quad (46)$$

The commutator is a c-number independent of the initial state of the field. Many authors in quantum optics claim that this contribution has no great physical interest. So the matrix elements determining the detection of quanta of the field are of the form

$$\langle 0, M | \varphi^{(-)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) | 0, M \rangle + \langle 0, M | \varphi^{(-)}(\eta, \xi) \varphi^{(+)}(\eta', \xi') | 0, M \rangle. \quad (47)$$

Substituting the modes given by eq.(22) in eq.(44) it is possible to show that the rotating detector has non-zero probability of excitation. Since the contribution given by eq.(47) is zero (there are no rotating particles in Minkowski vacuum), the non-zero rate is caused by the last term in eq.(46). A disagreeable situation emerges. Our apparatus device is not measuring the particle content of some state.

The first solution of the puzzle of the rotating detector was given a few months ago by Davies, Dray and Manogue [1]. These authors assumed that the field is defined only in the interior of a cylinder of radius R in such a way that the rotating Killing vector $\partial_t - \Omega \partial_\theta$ is always timelike. Consequently the response function of the rotating detector is zero. Of course if the angular velocity of the detector is above some threshold, excitation occurs. Clearly the excitation of the rotating detector is related with the mistake of use a Galilean transformation for the introduction of coordinates in the rotating frame.

Let us assume that a uniformly rotating frame must be a Lorentz frame. In this situation a naturally adapted coordinate system to a such frame is the one defined by Trocherries and Takeno [19, 20]. The advantage of this coordinate system is that the velocity of a rotating point is $v = \tanh \Omega r$ (for small radius or angular velocities we recovered the situation $v = \Omega r$). This adapted coordinate system cover all the Minkowski manifold for all angular velocities. Although we will be not able to calculate explicitly the Bogoliubov coefficients between the inertial and the rotating modes we will prove that these coefficients are non-zero and in this case the answer obtained calculating the Bogoliubov coefficients between cartesian and rotating modes and the response function of the detector will agree.

To prove the above assumption, first we have to canonical quantize a massless scalar field using this adapted coordinate system given by eqs.(9-12). Making a Taylor expansion for $\cosh \Omega r$ and $\tanh \Omega r$ and retaining terms up the first order in Ωr the line element becomes

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - 4r\Omega \theta dr dt - dz^2. \quad (48)$$

It is important to stress that the metric given by Eq. (48) is an approximation to the original time dependent metric and the solutions that we obtain are approximations to the solutions for the modes in the true metric. Note that since we can not solve exactly the Klein-Gordon equation without the approximation we can not prove the above assumption. We believe that the approximation in the complete solution is equivalent to the exact solution of the Klein-Gordon equation on the low velocity limit³.

³We remark that the Riemann tensor derived using the line element given by Eq. (48) is null in the considered approximation order.

We point out that although we will consider only the case $\Omega r < 1$, the low-velocity limit of Takeno's transformation does not give the "Galilean" transformation since we have

$$t = t' - \Omega r'^2 \theta' \quad (49)$$

$$r = r', \quad (50)$$

$$\theta = \theta' - \Omega t', \quad (51)$$

$$z = z'. \quad (52)$$

Although the angular variables (θ, θ') are connected as Eq. (6) the time-like variables (t, t') has a peculiar transformation law. We found such kind of behavior in spinning cosmic string spacetime, with angular momentum per unit length J , with line element:

$$ds^2 = (dt + 4GJd\theta)^2 - dr^2 - b^2 r^2 d\theta^2 - dz^2. \quad (53)$$

G is the Newtonian constant and $b = 1 - 4G\mu$, where μ is the mass per unit length. This metric is locally flat, as can immediately be confirmed by changing the coordinates according:

$$\tau = t + 4GJ\theta \quad (54)$$

$$\psi = (1 - 4G\mu)\theta. \quad (55)$$

As was discussed by Deser et al [43] to preserve single-valuedness we must identify time which differ by $8\pi GJ$. This gives to the spacetime a time helical structure. The difference in both cases is that in Eq. (49) we have a peculiar dependence in r^2 instead GJ in the case of the spinning cosmic string. The possible implications of this fact is still obscure for us.

In the low-velocity approximation the metric given by Eq. (48) is stationary by not static. This means that although there is a timelike Killing vector field K , the spatial sections putting $t = \text{constant}$ are not orthogonal to the time lines putting r, θ and z constants, i.e., the Killing vector K is not orthogonal to the spatial section. This line element describe a physical situation in which world lines infinitesimally close to a given world line are spatially rotating with respect to this world line [44].

In this simplified case, the Klein-Gordon equation reads

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} - 4\Omega\theta \frac{\partial}{\partial t} - \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} - 4\Omega\theta r \frac{\partial^2}{\partial r \partial t} - \frac{\partial^2}{\partial z^2} \right) \varphi(t, r, \theta, z) = 0. \quad (56)$$

The solution can be found using partial separation of variables

$$\varphi(t, r, \theta, z) = T(t)Z(z)f(r, \theta). \quad (57)$$

Choosing

$$Z(z) = e^{ik_z z} \quad (58)$$

and

$$T(t) = e^{-i\omega t}, \quad (59)$$

and finally, defining $\omega^2 = k_z^2 + q^2$ we obtain the equation for $f(r, \theta)$

$$\left[\frac{\partial^2}{\partial r^2} + \left(\frac{1}{r} - 4i\omega\Omega\theta r \right) \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + (q^2 - 4i\omega\Omega\theta) \right] f(r, \theta) = 0. \quad (60)$$

The perturbative solution of this equation was derived in the appendix A and is given by

$$\begin{aligned} f_\mu(y, \theta) &= C e^{i\mu\theta} \left[J_\mu(y) + l e^{i\lambda\theta} J_{\mu+\lambda}(y) \right] \\ &+ \frac{l}{2} \int d\theta' \int dy' G(y, \theta; y', \theta') \theta' \left[y'^3 J_{\mu-1}(y') + 2y'^2 J_\mu(y') - y'^3 J_{\mu+1}(y') \right]. \end{aligned} \quad (61)$$

where l , y and $G(y, \theta; y', \theta')$ are also defined in the appendix A, and C is a normalization factor.

We must now turn to the question of single valuedness of $f_\mu(y, \theta)$. This situation is very similar to the $(2+1)$ dimensional gravity [43, 45]. In our situation we have two different possibilities: the first is to assume that $f_\mu(y, \theta)$ is a single value function. When θ increases from 0 to 2π for a constant y , t jumps by Ωr^2 given a time helical structure. It is possible to show that this solution acquire a phase $e^{i\omega\Omega r^2}$. Using Dirac's arguments and also the Mazur ideas [46] the energy of the modes must be quantized

$$\omega = n \left(\frac{2\pi}{\Omega r^2} \right), \quad n = \text{integer}.$$

Note that at principle we can choose that Ω is quantized to eliminate the phase problem. With this choice the Bogoliubov coefficients $\beta_{k\nu}$ are zero. The second one is do not assume that $f_\mu(y, \theta)$ is a single value function, and in this case the Bogoliubov coefficients $\beta_{k\nu}$ are different from zero and proportional to $\omega\Omega^2 r^2$. It is important to realize that if we assume that $f_\mu(y, \theta)$ is single value the Minkowski and the rotating vacuum are the same only in the small velocity approximation. If we go further retaining terms of order $\Omega^2 r^2$ the Bogoliubov coefficients between the inertial and non inertial modes must be different from zero.

Although the spatial part of the solution of eq.(56) is extremely complicated, there is not ambiguity in the definition of positive and negative rotating modes since the temporal part is given by eq.(59) and the world line of the detector is an integral curve of the Killing vector $K = \partial/\partial t$ that generates a one-parameter group of isometries.

We have problems to define the Hamiltonian in the rotating frame if we work with the Takeno coordinate transformation without assume $\Omega r < 1$. The metric given by eqs.(13-17) is not invariant under time translations. Usually the Hamiltonian is defined as

$$H = \int T^{\mu\nu} \xi_\mu d\sigma_\nu \sqrt{-g} \quad (62)$$

where ξ_μ is a timelike Killing vector field. Since in the rotating frame the line element is not stationary it is a complicated question how to define H_R . A possible solution of this problem is to use the same idea that we use in expanding universes where there is no timelike Killing vector field. It is possible to introduce the definition of particles at each time. This procedure introduce the difficult of particle creation [47], and is against the spirit of the problem which we are trying to solve.

Before canonical quantize the massless scalar field we will put in a different way the problem of the rotating detector and the solution that we obtained for it. Let us accept that the the rotating frame is a Lorentz frame and the rotating disc has a unlimited size. In this case we must use the Takeno's coordinate system adapted to this rotating frame. We are faced with a very difficult problem, i.e., to solve the Klein-Gordon equation without any approximation. We adopted an approximation⁴ and obtained a result totally different from Davies et al. The Davies et al rotating vacuum is defined only in the interior of a cylinder and coincides with the Minkowski vacuum. Our result is also valid only in a limited region but with a important physical different result: the rotating vacuum is different from the Minkowski one. We recognise that the line element given by Eq. (48) put very difficult problems, it is a multivalued metric, etc. Presumably the correct solution is to solve the full unapproximated wave equation. In this case the observers would not move along the integral curves of a Killing vector field and the usual quantization procedure is problematic. Nevertheless it is possible to deal with this situation. For a careful study of this case see ref. [48]. If it is possible to implement the canonical quantization procedure in this non stationary situation the resulting rotating vacuum would be well described. In the absence of such solution we continue to use the low velocity approximation.

Going back to the low-limit velocity case, we have that the line element is stationary and there is no ambiguity to define the rotating vacuum $|0, R\rangle$ is such a way that:

$$b_{q\mu k_z}|0, R\rangle = 0. \quad \forall \quad q, \mu, k_z, \quad (63)$$

where

$$\varphi(t, r, \theta, z) = \sum_{\mu} \int q dq dk_z \left[b_{q\mu k_z} v_{q\mu k_z}(t, r, \theta, z) + b_{q\mu k_z}^{\dagger} v_{q\mu k_z}^*(t, r, \theta, z) \right]. \quad (64)$$

By sake of simplicity let use the following notation:

$$\varphi(t, r, \theta, z) = \sum_{\nu} b_{\nu} v_{\nu}(t, r, \theta, z) + b_{\nu}^{\dagger} v_{\nu}^*(t, r, \theta, z), \quad (65)$$

where $\nu \equiv \{q, \mu, k_z\}$ is a collective index.

It is straightforward to show that the Minkowski vacuum can be expressed as a many rotating-particles state. By comparing the expansion of the field operator using the inertial modes and the rotating modes it is possible to obtain the expression comparing both vacua, i.e $|0, M\rangle$ and $|0, R\rangle$:

$$|0, M\rangle = e^{\frac{i}{2} \sum_{\mu, \nu} b^{\dagger}(\mu) V(\mu, \nu) b^{\dagger}(\nu)} |0, R\rangle \quad (66)$$

where

$$V(\mu, \nu) = i \sum_k \beta_{\mu k}^* \alpha_{k\nu}^{-1}, \quad (67)$$

and the Bogoliubov coefficients are given by $\beta_{\nu k} = -(v_{\nu}, \psi_k^*)$ and $\alpha_{\nu k} = (v_{\nu}, \psi_k)$. It is clear that the number of rotating particles in a specific mode in the Minkowski vacuum is given by

$$\langle 0, M | N_R(\nu) | 0, M \rangle = \sum_k |\beta_{k\nu}|^2. \quad (68)$$

⁴Actually we are in the same level as Davies et al solution, since we considered only the low velocity limit.

Let us choose the hypersurface $t' = \text{constant}$ to find the Bogoliubov coefficients, i.e.,

$$\beta_{\nu k} = i \int_0^{2\pi} d\theta' \int_{-\infty}^{\infty} dz \int_0^{\infty} r dr \{v_{\nu}(x') [\partial_{t'} \psi_k(x')] - [\partial_{t'} v_{\nu}(x')] \psi_k(x')\}. \quad (69)$$

The Bogoliubov coefficients $\beta_{\nu k}$ must be non-zero since the positive and negative frequency rotating modes are mixture between positive and negative inertial modes. The important conclusion from the above arguments is that the Minkowski and this rotating vacuum are not the same. Now we will show that the expectation value of the number operator of rotating particles is proportional to the response function, recovering the old idea that rate of excitation is proportional to the number of particles (in the mode of interest) in the state of the field. Note that the rate given by eq.(45) can be written as:

$$R(E, \Delta T, \xi, \xi') = R_I(E, \Delta T, \xi, \xi') + R_{II}(E, \Delta T, \xi, \xi') \quad (70)$$

where

$$R_I(E, \Delta T, \xi, \xi') = \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \left(\langle 0, M | \varphi^{(-)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) | 0, M \rangle + \langle 0, M | \varphi^{(-)}(\eta, \xi) \varphi^{(+)}(\eta', \xi') | 0, M \rangle \right) \quad (71)$$

and

$$R_{II}(E, \Delta T, \xi, \xi') = \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \langle 0, M | [\varphi^{+}(\eta', \xi'), \varphi^{-}(\eta, \xi)] | 0, M \rangle. \quad (72)$$

Let us investigate the contribution given by Eqs. (71) and (72) to the rate assuming the detector at rest in the rotating frame. Substituting Eqs. (58), (59) and (61) in Eq. (72) we have for $R_{II}(E, \Delta T, \xi)$

$$R_{II}(E, \Delta T, \xi) = \sum_{\mu\nu} \delta_{\mu\nu} |Z|^2 f_{\mu}(\xi) f_{\nu}^*(\xi) \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \left[e^{-i\omega_{\mu}(\eta - \frac{\omega_{\mu}}{\omega_{\nu}} \eta')} + e^{i\omega_{\nu}(\eta - \frac{\omega_{\mu}}{\omega_{\nu}} \eta')} \right]. \quad (73)$$

It is not difficult to perform the ζ integral and obtain

$$R_{II}(E, \Delta T, \xi) = \Delta T \sum_{\mu} |Z|^2 |f_{\mu}(\xi)|^2 \left\{ \frac{\sin[(E - \omega_{\mu})\Delta T]}{(E - \omega_{\mu})\Delta T} + \frac{\sin[(E + \omega_{\mu})\Delta T]}{(E + \omega_{\mu})\Delta T} \right\}. \quad (74)$$

In the asymptotic limit we have

$$\lim_{\Delta T \rightarrow \infty} R_{II}(E, \Delta T, \xi) = \sum_{\mu} |Z|^2 |f_{\mu}(\xi)|^2 (\delta(E - \omega_{\mu}) + \delta(E + \omega_{\mu})). \quad (75)$$

Note that in $R_{II}(E, \Delta T, \xi)$ we have two processes: absorption and emission processes of rotating particles. This contribution to the rate does not depend on the number of rotating particles in the Minkowski vacuum. The $R_{II}(E, \Delta T, \xi)$ contribution is independent of the particle content of the state of the field. This is the reason that Glauber and others disregarded the $R_{II}(E, \Delta T, \xi)$ term to the rate. It is important to have in mind although $R_{II}(E, \Delta T, \xi)$ is not related with the particle content of the state, it is responsible for the spontaneous decay process. To have a better understanding of the meaning

of $R_{II}(E, \Delta T, \xi)$ let us repeat the calculations preparing the field in the rotating vacuum i.e.

$$R_{II}(E, \Delta T, \xi, \xi') = \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \langle 0, R | [\varphi^+(\eta', \xi'), \varphi^-(\eta, \xi)] | 0, R \rangle. \quad (76)$$

In the case of spontaneous excitation ($E > 0$) the asymptotic limit for eq.(76) gives zero (stability of the detector's ground state). For the rotating detector interacting with the field in the Minkowski vacuum, we will have this term plus a term proportional to $\Omega^2 r^2$, i.e., for the case of spontaneous decay we have

$$\lim_{\Delta T \rightarrow \infty} R_{II}(E, \Delta T) = \frac{|E|}{2\pi} (1 - 16\Omega^2 r^2) \quad (77)$$

The probability of decay per unit time decrease with the square of the distance from the origin. Since we are using the approximation $\Omega^2 r^2 < 1$, Eq. (76) never becomes negative. Let us analyze the contribution given by $R_I(E, \Delta T, \xi)$. It is not difficult to show that the term between the parenthesis in eq.(71) gives

$$\begin{aligned} \langle 0, M | \varphi^{(-)}(\eta', \xi') \varphi^{(+)}(\eta, \xi) | 0, M \rangle + \langle 0, M | \varphi^{(-)}(\eta, \xi) \varphi^{(+)}(\eta', \xi') | 0, M \rangle = \\ \sum_{\mu\nu} \sum_k \beta_{\nu k}^* \beta_{k\mu} \left[v_\nu(\eta', \xi') v_\mu^*(\eta, \xi) + v_\nu(\eta, \xi) v_\mu^*(\eta', \xi') \right]. \end{aligned} \quad (78)$$

Consequently the $R_I(E, \Delta T, \xi)$ contribution to the rate of transition for the detector at rest in the rotating frame is:

$$R_I(E, \Delta T, \xi) = \sum_{\mu\nu} \sum_k \beta_{\mu k}^* \beta_{k\nu} |Z|^2 f_\mu(\xi) f_\nu^*(\xi) \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \left[e^{-i\omega_\mu(\eta - \frac{\omega_\nu}{\omega_\mu}\eta')} + e^{i\omega_\nu(\eta - \frac{\omega_\mu}{\omega_\nu}\eta')} \right]. \quad (79)$$

In this expression there are two different contributions: the non-diagonal and the diagonal terms ($\mu = \nu$). Let us analyze the contribution to the rate from the diagonal terms given by

$$R_I(E, \Delta T, \xi) = \sum_\mu \sum_k \beta_{\mu k}^* \beta_{k\mu} |Z|^2 f_\mu(\xi) f_\mu^*(\xi) \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \left[e^{-i\omega_\mu(\eta - \eta')} + e^{i\omega_\mu(\eta - \eta')} \right]. \quad (80)$$

It is not difficult to perform the ζ integral and using eq.(68) we obtain

$$\begin{aligned} R_I(E, \Delta T, \xi) = \Delta T \sum_\mu |Z|^2 |f_\mu(\xi)|^2 \langle 0, M | N_R(\mu) | 0, M \rangle \\ \times \left\{ \frac{\sin[(E - \omega_\mu)\Delta T]}{(E - \omega_\mu)\Delta T} + \frac{\sin[(E + \omega_\mu)\Delta T]}{(E + \omega_\mu)\Delta T} \right\} \end{aligned} \quad (81)$$

In the asymptotic limit the rate of transition becomes

$$\lim_{\Delta T \rightarrow \infty} R_I(E, \Delta T, \xi) = \sum_\mu |Z|^2 |f_\mu(\xi)|^2 \langle 0, M | N_R(\mu) | 0, M \rangle (\delta(E - \omega_\mu) + \delta(E + \omega_\mu)). \quad (82)$$

We have two different processes in the above expression: absorption of rotating particles ($E > 0$) and induced emission of rotating particles ($E < 0$). In both cases the rate of transition will be proportional to the number of rotating particles with energy E in the

Minkowski vacuum multiplied by the square of the “wavefunction” ($|Z||f(r, \theta)|$) in the world line of the detector. This situation is exactly the case of a detector in a thermal bath where the rate of absorption of the particles of the bath is equal to the rate of induced emission. Note that we still have to the rate of absorption and emission processes the contribution to the non-diagonal terms.

Bell and Leinaas studied the depolarization problem in accelerators trying to use the idea of a Unruh-Davies effect. The electron in a accelerated ring is a magnetic version of the monopole detector, since there is a linear coupling between the magnetic field B and the magnetic moment of the electron. To see this result let us define the invariant operator

$$H = \frac{e}{2m^2} F_{\mu\nu}^* p^\mu s^\nu \quad (83)$$

where m is the electron mass, $F_{\mu\nu}^* = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ and s^ν is the four vector spin operator. In the frame in which the electron is at rest the operator H describes the interaction between the spin magnetic moment of the electron with the magnetic field,

$$H = -\vec{\mu} \cdot \vec{B}. \quad (84)$$

To understand the depolarization problem let us suppose an ensemble of detectors in equilibrium with a thermal bath. The probability to find the detector in the state $|i\rangle$ is:

$$P_i = \frac{e^{-\beta\omega_i}}{Z} \quad (85)$$

or

$$\frac{P_e}{P_g} = e^{-\beta E} \quad (86)$$

defining the occupation number $N(e)$ and $N(g)$ we have

$$N(e) = N(g)e^{-\beta E}. \quad (87)$$

Since the electron in a accelerator is a magnetic version of the monopole detector, in the equilibrium the rate between spin up and spin down will be given by the above equation. Thus if we introduce a complete unpolarized electron beam, it will suffer a polarization until the equilibrium is reached. The asymptotic rate of spin flip will be proportional to the asymptotic limit of the rate $R_\beta(E, \Delta T)$ i.e.,

$$\lim_{\Delta T \rightarrow \infty} R_\beta(E, \Delta T) = \frac{|E|}{2\pi} \left[\Theta(-E) \left(1 + \frac{1}{e^{\beta|E|} - 1} \right) + \Theta(E) \frac{1}{e^{\beta E} - 1} \right]. \quad (88)$$

Note that although the situation is similar to the Rindler’s case where the detector goes to excited state by absorption of Rindler’s particles (the Minkowski vacuum is a thermal state of Rindler’s particles), there is a fundamental difference. In the Rindler’s case there is an past and future horizon. Part of information which would have an inertial observer is inaccessible for accelerated observers. Although the Minkowski vacuum $|0, M\rangle$ is a pure state, for accelerated observers it must be described by a statistical operator. This is the origin of the thermal distribution of particles. As was noted by Bell and Leinaas in the case of circular motion the measurements of the polarization *does not agree* with

the calculations if we interpret the polarization by thermal effects. In our approach, depolarization is related with the fact that the Minkowski vacuum is a many particle state of rotating particles with a non thermal spectrum. Let us try to improve this ideas using Einstein's arguments [49]. All calculations will be held in the rotating frame. Suppose that the probability to find the detector in the state $|i\rangle$ is given by

$$P(\omega_i) = \frac{f(\omega_i)}{Z} \quad (89)$$

where the partition function Z is given by

$$Z = \sum_{i=1}^2 f(\omega_i). \quad (90)$$

Still following Einstein's arguments we have three different processes: absorption of rotating particles, induced emission and spontaneous emission (stimulated emission by the $|0, R\rangle$ vacuum fluctuations) of rotating particles. Defining the rate of spontaneous decay by $A_{2\rightarrow 1}(E, \Delta T)$ we have

$$dW_{2\rightarrow 1}(E, \Delta T) = A_{2\rightarrow 1}(E, \Delta T)dt \quad (91)$$

For the induced emission $R_{2\rightarrow 1}(E, \Delta T)$ we have

$$dW_{2\rightarrow 1}(E, \Delta T) = R_{2\rightarrow 1}(E, \Delta T)dt, \quad (92)$$

and finally for the rate of absorption $R_{1\rightarrow 2}(E, \Delta T)$ we have

$$dW_{1\rightarrow 2}(E, \Delta T) = R_{1\rightarrow 2}(E, \Delta T)dt. \quad (93)$$

In the *equilibrium situation* between an ensemble of rotating detectors and the scalar field in the Minkowski vacuum (asymptotic limit) we have

$$f(\omega_1)\rho(E)R_{1\rightarrow 2}(E) = f(\omega_2)[\rho(E)R_{2\rightarrow 1}(E) + A_{2\rightarrow 1}(E)] \quad (94)$$

where ρ is the number of rotating particle in the mode E in the Minkowski vacuum i.e.

$$\rho(E) = \langle 0, M | N_R(E) | 0, M \rangle. \quad (95)$$

Although the spectrum of the rotating particles in the Minkowski vacuum is not known, at the equilibrium we have $R_{1\rightarrow 2}(E) = R_{2\rightarrow 1}(E)$. In the equilibrium situation this hypothesis must hold. Note that this is not in principle fundamental for our conclusions. A straightforward calculations gives

$$\rho(E) = \frac{A_{2\rightarrow 1}(E)}{R_{1\rightarrow 2}(E)} \frac{1}{\frac{f(\omega_1)}{f(\omega_2)} - 1} \quad (96)$$

The knowledge of the Bogoliubov coefficients $\beta_{k\mu}$ give us both $\rho(E)$ and $R_{2\rightarrow 1}(E)$. A second step in our analysis is to use the result that $A_{2\rightarrow 1}(E)$ is exactly the rate of spontaneous decay of a inertial detector interacting with the field in the Minkowski vacuum. Thus we have

$$\frac{f(\omega_1)}{f(\omega_2)} = \left(\frac{ER_{1\rightarrow 2}^{-1}(E)}{\langle 0, M | N_R(E) | 0, M \rangle} - 1 \right). \quad (97)$$

This result show us the connection between the the rate between up and down spins as a function of the mean life of the excited state and $\rho(E)$ after the equilibrium situation is reached.

We still have to answer some questions. Where does the energy of excitation come from if we analyse the process from the point of view of the inertial observer? The non-inertial observer does not meet any difficulty. At some initial time we prepare the detector in the ground state and the field in the Minkowski vacuum. Since the Minkowski vacuum is a many rotating-particles state the detector goes to excited state absorbing a positive energy particle. For large time intervals energy conservation holds. For the point of view of the inertial observer the field is in the empty state. How is possible the excitation? A natural answer is to say that it is necessary an external accelerating agency to supply energy to keep the detector in the rotating world-line. It is possible to show that the detector goes to excited state with the emission of a Minkowski particle. In the next section we will perform the second quantization of the detector Hamiltonian to analyse the absorption and emission processes from the inertial point of view.

4 Second Quantization of the Total Hamiltonian and Polarization Effects on Electrons and Positrons in Storage Rings

In this section we will prove that the process: absorption (emission) of positive energy rotating particle with excitation (decay) of the detector (from the non-inertial point of view) is interpreted as a emission of a Minkowski particle with excitation (decay) of the detector from the inertial point of view. This simple result express the fact that electrons (positrons) experience a gradual polarization orbiting in a storage ring. This mechanism lead to the emission of spin-flip synchronon radiation [50]. It is important to stress that the amount of spin-flip radiation is extremely small compared with the non-flip radiation. An open question is why the polarization is not complete after the system reach the equilibrium? We will try to answer this question applying the ideas developed by us in the preceding sections. Of course again we have a oversimplified description of the phenomenon. Before start the second quantization of the detector and interaction Hamiltonian let us remember the fundamental results of the preceeding section (we will use a different notation in this section).

In Minkowski space time it is possible to quantize a massless scalar field using the cartesian coordinate adapted to inertial observers. Thus the scalar field can be expanded using an orthonormal set of modes

$$\varphi(x) = \sum_i a_i u_i(x) + a_i^\dagger u_i^*(x) \quad (98)$$

where

$$a_i |0, M \rangle = 0 \quad \forall i. \quad (99)$$

A rotating observer can also second quantize the scalar field and this quantization is unitarily non-equivalent to the quantization implemented by inertial observers. Thus the

scalar field can be expanded using a second set of orthonormal modes

$$\varphi(x) = \sum_j b_j v_j(x) + b_j^\dagger v_j^*(x) \quad (100)$$

where

$$b_j |0, R\rangle = 0 \quad \forall j. \quad (101)$$

As both sets are complete, the non-inertial modes can be expanded in terms of the inertial ones, i.e.

$$v_j(x) = \sum_i \alpha_{ji} u_i(x) + \beta_{ji} u_i^*(x) \quad (102)$$

or

$$u_i(x) = \sum_j \alpha_{ji}^* v_j(x) - \beta_{ji} v_j^*(x). \quad (103)$$

Using these equations and the orthonormality of the modes it is possible to write the annihilation and creation operators of inertial particles in the mode i as a linear combination of non-inertial creation and annihilation operators [51], i.e.

$$a_i = \sum_j \alpha_{ji} b_j + \beta_{ji}^* b_j^\dagger \quad (104)$$

or

$$b_j = \sum_i \alpha_{ji}^* a_i - \beta_{ji}^* a_i^\dagger. \quad (105)$$

Let us use the notation introduced in section 3, i.e. $|g\rangle = |1\rangle$ and $|e\rangle = |2\rangle$. Thus we have

$$H_D |i\rangle = \omega_i |i\rangle \quad i = 1, 2. \quad (106)$$

Using the above equation and the orthonormality of the energy eigenstates of the detector Hamiltonian, we can write

$$H_D = \sum_{i=1}^2 \omega_i |i\rangle \langle i|. \quad (107)$$

To second quantize the detector Hamiltonian we have to introduce the Dicke operators [52]

$$S^+ = |2\rangle \langle 1|, \quad (108)$$

$$S^- = |1\rangle \langle 2|, \quad (109)$$

and finally

$$S_z = \frac{1}{2}(|2\rangle \langle 2| - |1\rangle \langle 1|). \quad (110)$$

In the case of n eigenstates of the (atom) detector Hamiltonian we have to work with the atomic operators, i.e. the multilevel generalization of the Dicke spin operators for the two level system. The detector Hamiltonian in the two level case can be written as

$$H_D = E S_z + \frac{1}{2}(\omega_1 + \omega_2). \quad (111)$$

The operators S^+ , S^- and S_z satisfy the angular momentum commutation relations corresponding to spin 1/2 value, i.e.

$$[S^+, S^-] = 2S_z, \quad (112)$$

$$[S_z, S^+] = S^+, \quad (113)$$

$$[S_z, S^-] = -S^-. \quad (114)$$

It is clear that S^+ and S^- are respectively raising and lowering operators of the detector states ($S^+|1\rangle = |2\rangle$, $S^+|2\rangle = 0$, $S^-|2\rangle = |1\rangle$, $S^-|1\rangle = 0$). The interaction Hamiltonian given by eq.(27) can be written as

$$H_{int} = \lambda[m_{21}S^+ + m_{12}S^- + S_z(m_{22} - m_{11})]\varphi(x), \quad (115)$$

where

$$\langle i|m(0)|j\rangle = m_{ij}, \quad (116)$$

and $\varphi(x)$ must be evaluated in the world line of the detector. We should simplify the discussion choosing $m_{11} = m_{22}$. As we will see the part of the interaction hamiltonian with the S_z operator is responsible for the non-flip synchrotron radiation. Substituting eq.(100) in eq.(115) we see that there are different processes with absorption or emission of rotating particles with excitation or decay of the detector. It is possible to show that some of these processes are virtual, and only processes with energy conservation survive in the asymptotic limit, i.e., excitation of the detector with absorption of a rotating particle (process involving $b_j S^+$) and decay of the detector with emission of a rotating particle (process involving $b_j^\dagger S^-$).

The first process is generated by the following operators:

$$m_{12} \sum_j v_j(x) b_j S^+. \quad (117)$$

Substituting eq.(102) and eq.(105) in eq.(117) it is clear that the above process of absorption of a rotating particle in the mode j is the following:

$$\sum_{ijk} [\beta_{ji}^* \alpha_{jk} u_k(x) + \beta_{ji}^* \beta_{jk} u_k^*(x)] a_i^\dagger S^+. \quad (118)$$

Therefore this process for the inertial observer is an excitation of the detector with creation of Minkowski particles.

The second process is generated by the following operators:

$$m_{21} \sum_j v_j^*(x) b_j^\dagger S^-. \quad (119)$$

Substituting eq.(102) and eq.(105) in eq.(119) we see that the above process of emission of a rotating particle in the mode j is the following:

$$\sum_{ijk} [\alpha_{ij} \alpha_{jk}^* u_k^*(x) + \alpha_{ij} \beta_{jk}^* u_k(x)] a_i^\dagger S^-. \quad (120)$$

Therefore this process for the inertial observer is a decay of the detector with creation of Minkowski particles.

Now we are able to understand the problem of the synchrotron radiation. In the emission of synchrotron radiation by electrons moving along a circular orbit, there are two kinds of processes: the first is the emission of photons without spin flip of the electron and the second is emission with spin flip. We will restrict our discussion to the second case. To make a parallel with the detector's problem we have to assume that the electron trajectory is "classical" (there is no fluctuation of the electron path) or even after the photon emission there is no recoil (as was stressed by Bell and Leinaas, the results does not depend on position fluctuations of the electron trajectory). There are two different results in the literature depending on the value of the Landé-g factor of the electron. Jackson showed that the rate of transition from an initial state with the spin of the electron directed along the magnetic field (high energy state) to a state with the electron spin in opposite to the magnetic field (lower energy state) is lower than the opposite situation if the Landé-g factor of the electron obeys $0 < g < 1.2$. It is important to stress that the situation is opposite of the naive description where polarization arises from spontaneous emission as the spin move from its "upper" (high energy state) to its "lower" (low energy state) in the magnetic field. For the case where $1.2 < g < 2$ Jackson and also Sokolov et al [53] obtained that after the photon emission the electron spin will tends to orient themselves in opposite to the magnetic field (going to the lower energy state). Of course, positrons spins will have an opposite behavior. These both results are consistent with our interpretation that absorption (emission) of a rotating particle with excitation (decay) of the detector in the non-inertial frame is interpreted as emission of a Minkowski particle with excitation (decay) of the detector in the inertial frame.

To find the degree of polarization before the equilibrium situation is achieved let us define the occupation number of electrons with spins directed in opposition to the magnetic field (lower energy state) by N_1 , and N_2 is the number of electrons with spins directed to the magnetic field. Of course we have $N_1(t) + N_2(t) = N$, where $N = \text{constant}$ is the total numbers of electrons in the ring. We will do all the calculations in the rotating frame. The degree of polarization of an ensemble of electrons in the beam is defined as

$$P(t) = \frac{N_1(t) - N_2(t)}{N_1(t) + N_2(t)}. \quad (121)$$

The equation of the evolution of N_1 and N_2 are given by

$$\frac{dN_1}{dt} = N_2 [\rho(E)R_{2 \rightarrow 1}(E, \Delta T) + A_{2 \rightarrow 1}(E, \Delta T)] - N_1 [\rho(E)R_{1 \rightarrow 2}(E, \Delta T)] \quad (122)$$

and

$$\frac{dN_2}{dt} = N_1 [\rho(E)R_{1 \rightarrow 2}(E, \Delta T)] - N_2 [\rho(E)R_{2 \rightarrow 1}(E, \Delta T) + A_{2 \rightarrow 1}(E, \Delta T)] \quad (123)$$

Let us avoid the difficult to find $R_{1 \rightarrow 2}(E, \Delta T)$ and $R_{2 \rightarrow 1}(E, \Delta T)$ and using the following approximation, i.e.,

$$\rho(E)R_{2 \rightarrow 1}(E, \Delta T) + A_{2 \rightarrow 1}(E, \Delta T) = \sigma_{21} = \text{constant} \quad (124)$$

and

$$\rho(E)R_{1\rightarrow 2}(E, \Delta T) = \sigma_{12} = \text{constant}. \quad (125)$$

We are choosing the asymptotic limit (see eq.(75) and eq.(82)). Then, starting from a situation where there is no polarization, i.e., $P(t = 0) = 0$ it is possible to find the polarization until the equilibrium situation is achieved. It is necessary only to integrate the above equations. A straightforward calculation gives

$$N_1(t) = \frac{N}{2} \left(\frac{\sigma_{12} - \sigma_{21}}{\sigma_{12} + \sigma_{21}} \right) e^{-(\sigma_{12} + \sigma_{21})t} + N \left(\frac{\sigma_{21}}{\sigma_{12} + \sigma_{21}} \right) \quad (126)$$

and

$$N_2(t) = -\frac{N}{2} \left(\frac{\sigma_{12} - \sigma_{21}}{\sigma_{12} + \sigma_{21}} \right) e^{-(\sigma_{12} + \sigma_{21})t} + N \left(\frac{\sigma_{12}}{\sigma_{12} + \sigma_{21}} \right). \quad (127)$$

The degree of polarization of the beam is

$$P(t) = \left(\frac{\sigma_{21} - \sigma_{12}}{\sigma_{12} + \sigma_{21}} \right) \left(1 - e^{-(\sigma_{12} + \sigma_{21})t} \right). \quad (128)$$

We obtained that if $R_{1\rightarrow 2}(E, \Delta T)$, $R_{2\rightarrow 1}(E, \Delta T)$ and $A_{2\rightarrow 1}(E, \Delta T)$ are independent of time the asymptotic degree of polarization is constant i.e.,

$$\lim_{t \rightarrow \infty} P(t) = \left(\frac{\sigma_{21} - \sigma_{12}}{\sigma_{12} + \sigma_{21}} \right). \quad (129)$$

Experimental results show us a not complete polarization. Why there is residual depolarization? This is the puzzle stressed by Jackson [50] and also Bell and Leinas [2]. From the former equation it is easy to see that the polarization can not be complete. The process absorption of a rotating particle with excitation of the detector has always non null probability. In the asymptotic limit we have that if

$$R_{21} + A_{21} > 3R_{12}, \quad (130)$$

the lower energy state is preferred ($1.2 < g < 2$, for the Landé-g factor), and if

$$R_{21} + A_{21} < 3R_{12}, \quad (131)$$

the higher energy state is preferred ($0 < g < 1.2$ for the Landé-g factor).

We remark that the results that the polarization can not be complete was obtained in a very crude approximation where the rates $R_{1\rightarrow 2}(E, \Delta T)$, $R_{2\rightarrow 1}(E, \Delta T)$ and $A_{2\rightarrow 1}(E, \Delta T)$ does not depend on time [see eq.(74) and eq.(81)]. A more realistic result can be obtained assuming that this rates does depend on time. Defining $n_1 = N_1/N$ and $n_2 = N_2/N$ and also

$$\rho(E)R_{2\rightarrow 1}(E, \Delta T) + A_{2\rightarrow 1}(E, \Delta T) = \sigma_{21}(t) \quad (132)$$

and

$$\rho(E)R_{1\rightarrow 2}(E, \Delta T) = \sigma_{12}(t) \quad (133)$$

we obtain the following equations:

$$n_1(t) + n_2(t) = 1 \quad (134)$$

and

$$\frac{dn_1(t)}{dt} + n_1(t) [\sigma_{12}(t) + \sigma_{21}(t)] = \sigma_{21}(t) \quad (135)$$

Let us consider the homogeneous linear equation:

$$\frac{dn_1^{(0)}(t)}{dt} = n_1^{(0)}(t) [\sigma_{12}(t) + \sigma_{21}(t)] = 0 \quad (136)$$

A general solution is

$$n_1^{(0)}(t) = C_1 e^{-\int^t [\sigma_{12}(t') + \sigma_{21}(t')] dt'} \quad (137)$$

Now let us substitute in the non-homogeneous equation the expression

$$n_1(t) = v(t) e^{-\int^t [\sigma_{12}(t') + \sigma_{21}(t')] dt'} \quad (138)$$

The equation for $v(t)$ becomes

$$\frac{dv(t)}{dt} e^{-\int_0^t [\sigma_{12}(t') + \sigma_{21}(t')] dt'} = \sigma_{21}(t) \quad (139)$$

consequently we have

$$v(t) = C_2 + \int^t dt' \sigma_{21}(t') e^{\int^{t'} [\sigma_{12}(t'') + \sigma_{21}(t'')] dt''} \quad (140)$$

The general solution that we are looking for involves two quadratures and it is given by

$$n_1(t) = C_2 e^{-\int^t [\sigma_{12}(t') + \sigma_{21}(t')] dt'} + e^{-\int^t [\sigma_{12}(t') + \sigma_{21}(t')] dt'} \int^t dt' \sigma_{21}(t') e^{\int^{t'} [\sigma_{12}(t'') + \sigma_{21}(t'')] dt''} \quad (141)$$

With the values of $R_{2 \rightarrow 1}(E, \Delta T)$, $R_{1 \rightarrow 2}(E, \Delta T)$ and $A_{2 \rightarrow 1}(E, \Delta T)$ (given by eqs.(74) and (81)) it is possible to find the degree of polarization.

We would like to point out that there is a different approach to study these problems. As it has been pointed out by Milonni and Smith [54] and Ackerhalt, Knight and Eberly [55], it is possible to study radiative processes without using perturbation theory, but using the Heisenberg equations of motion. In this approach it is possible to obtain non-perturbative expressions for radiative processes where the radiation reaction appears in a very simple way: the part of the field due to the atom (detector) that drives the Dicke operators [52]. In this approach it is possible to identify the role of radiation reaction and vacuum fluctuations in spontaneous emission. We would like to stress the fact that the contribution of vacuum fluctuations and radiation reaction can be chosen arbitrarily, depending on the order of the Dicke and field operators. As it was discussed by Dalibard, Dupont-Roc and Cohen-Tannoudji [56], there is a preferred ordering in such a way that the vacuum fluctuations and radiation reaction contribute equally to the spontaneous emission process. More recently this approach was developed by Audretsch and Muller, and also Audretsch, Muller and Holzmann [57] to study the Unruh-Davies effect. These authors constructed the following picture of the Unruh-Davies effect. The effect of vacuum fluctuations is changed by the acceleration, although the contribution of radiation reaction is unaltered. Due to the modified vacuum fluctuation contribution, transition to an excited state becomes possible even in the vacuum. It will be interesting to use this formalism to study the rotating detector.

5 Conclusions

In this paper we discuss the relativistic problem of uniform rotation and how this question is related with the puzzle of the rotating detector. After this we discuss the response function of a particle detector traveling in different world lines interacting with a scalar field prepared in two different vacua: the Minkowski and the rotating vacuum. For electrons in storage rings, a residual depolarization has been found experimentally. Bell and Leinaas investigate this effect using the idea of circular Unruh-Davies effect. We propose an alternative solution to the rotating detector puzzle and how this will be related with depolarization effects in circular accelerators.

Let us use the result that the probability of transition per unit proper time depends not only of the world line of the “atom” but also the particular vacuum in which we prepare the field to study four different situations:

- i) The response function of an inertial detector interacting with the field in the Minkowski vacuum;
- ii) The response function of the rotating detector interacting with the field in the Minkowski vacuum;
- iii) The response function of an inertial detector interacting with the field in the rotating vacuum;
- iv) The response function of the rotating detector interacting with the field in the rotating vacuum.

The same kind of analysis in a different situation was given by Pinto Neto and Svaiter [58]. The case (i) gives the usual result that an inertial detector in its ground state interacting with the field in the Minkowski vacuum does not excite. It is clear that the situation (iv) will give the same result. The case (ii) can be produced in a laboratory. The case (iii) is more involved. How to produce the rotating vacuum? A possible solution is to use the ideas developed by Denardo and Percacci [28] and also Manogue [59]. This second author considers the case of rotating boundaries to push the vacuum around. Note that we are dealing with a Casimir rotating vacuum. Is it possible to create some kind of rotating vacuum? If the answer is positive we conjecture that the situation (iii) will give the same response function as situation (ii).

It will be of interest to explore the consequences of this paper, in particular to examine some interesting astrophysical situations. For example, the origin of non-thermal radio-frequency in the Universe can be explained by the mechanism of synchrotron radiation? [60]. Some authors discussed the metric of a spinning cosmic string [61]. We conjecture that electrons and positrons in the neighbourhood of such objects must emit synchrotron radiation. On the same grounds we conjecture that any rotating astrophysical object (spinning pulsars for example [62]) with a cloud of electrons and positrons is a source of synchrotron radiation. We can attempt to justify our conjecture using the well known result that the radiation emitted by a pulsar has a high degree of polarization. This fact suggests that the mechanism is similar to the one that generates synchrotron radiation.

Before finish we would like to make some comments concerning the Sagnac’s effect. This is the optical analogue of the Foucault pendulum. In the Sagnac’s experiment the apparatus device rotates, and the optical experiment can determine the rotation of the frame relative to an inertial frame. This shows the difference between inertial and

the rotating (non-inertial) frame. For inertial frames it is impossible to determine the absolute velocity of the apparatus. In the case of the rotating frame the angular velocity can be obtained. Our criticism of this scheme is the following: to measure the proper spatial line element (in the rotating frame) we have to measure the time taken by the light signal between an emission and also absorption from atoms. The connection with the detector puzzle shows how is intricate the analysis.

In conclusion, in this paper we have attempt to discuss the consequences of assuming that a rotating frame is a Lorentz frame of reference. If we second quantize a scalar field in this frame we show that, *the Minkowski and a rotating vacuum are not the same*. Although the Bogoliubov coefficients $\beta_{k\nu}$ between the inertial and the non-inertial modes are non-zero it is very difficult to calculate them. We are forced to admit that we fail to finish our interprize since we meet a basic difficulty to calculate the spectral density of rotating particles in the Minkowski vacuum. Is it possible to go further?

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Appendix A

In this appendix we will present the solution of Eq.(60):

$$\left[\frac{\partial^2}{\partial r^2} + \left(\frac{1}{r} - 4i\omega\Omega\theta r \right) \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + (q^2 - 4i\omega\Omega\theta) \right] f(r, \theta) = 0.$$

Let us define $g(r, \theta)$ by the following equation:

$$f_\mu(r, \theta) = e^{i\mu\theta} g_\mu(r, \theta).$$

A direct substitution gives the equation for $g(r, \theta)$:

$$\left[\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{\mu^2}{r^2} + q^2 \right) + \frac{1}{r^2} \left(\frac{\partial^2}{\partial \theta^2} + 2i\mu \frac{\partial}{\partial \theta} \right) - \sigma\theta \left(r \frac{\partial}{\partial r} + 1 \right) \right] g_\mu(r, \theta) = 0,$$

where $\sigma = 4i\omega\Omega$. Define the new quantity $y = qr$ and $l = \sigma/q^2$ the equation becomes

$$\left[\left(\frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} - \frac{\mu^2}{y^2} + 1 \right) + \frac{1}{y^2} \left(\frac{\partial^2}{\partial \theta^2} + 2i\mu \frac{\partial}{\partial \theta} \right) - l\theta \left(y \frac{\partial}{\partial y} + 1 \right) \right] g_\mu(y, \theta) = 0.$$

There appear to be no way of solve the above equation exactly. Consequently let us try a perturbative solution given by

$$g_\mu(y, \theta) = J_\mu(y) + \sum_{k=1}^{\infty} l^k P_\mu^{(k)}(y, \theta).$$

By considering only the first order term in the above expansion and for simplicity using the notation $P_\mu^{(1)}(y, \theta) \equiv P_\mu(y, \theta)$ we obtain:

$$\left(\frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} - \frac{\mu^2}{y^2} + 1 \right) P_\mu(y, \theta) + \frac{1}{y^2} \left(\frac{\partial^2}{\partial \theta^2} + 2i\mu \frac{\partial}{\partial \theta} \right) P_\mu(y, \theta) - \theta \left(y \frac{\partial}{\partial y} + 1 \right) J_\mu(y) = 0.$$

Defining

$$\frac{1}{2}y^3 J_{\mu-1}(y) + y^2 J_\mu(y) - \frac{1}{2}y^3 J_{\mu+1}(y) = h(y),$$

we get:

$$\left[\left(y^2 \frac{\partial^2}{\partial y^2} + y \frac{\partial}{\partial y} - \mu^2 + y^2 \right) + \frac{\partial^2}{\partial \theta^2} + 2i\mu \frac{\partial}{\partial \theta} \right] P_\mu(y, \theta) = \theta h(y).$$

It is possible to use the Green's functions method to find the general solution for $P_\mu(y, \theta)$. Thus,

$$P_\mu(y, \theta) = P_\mu^{(0)}(y, \theta) + \int d\theta' \int dy' G(y, \theta; y', \theta') \theta' h(y'),$$

where $P_\mu^{(0)}(y, \theta)$ is the solution of the homogeneous equation, and $G(y, \theta; y', \theta')$ satisfy

$$\left[\left(y^2 \frac{\partial^2}{\partial y^2} + y \frac{\partial}{\partial y} - \mu^2 + y^2 \right) + \frac{\partial^2}{\partial \theta^2} + 2i\mu \frac{\partial}{\partial \theta} \right] G(y, \theta; y', \theta') = \delta(y - y') \delta(\theta - \theta').$$

It is straightforward to find the solution of the homogeneous equation using separation of variables method defining:

$$P_\mu^{(0)}(y, \theta) = e^{i\lambda\theta} Q_\mu^{(0)}(y).$$

Then,

$$Q_\mu^{(0)}(y) = J_{\mu+\lambda}(y).$$

Finally the general solution is given by:

$$\begin{aligned} f_\mu(y, \theta) &= e^{i\mu\theta} \left[J_\mu(y) + l e^{i\lambda\theta} J_{\mu+\lambda}(y) \right] \\ &+ \frac{l}{2} \int d\theta' \int dy' G(y, \theta; y', \theta') \theta' \left[y'^3 J_{\mu-1}(y') + 2y'^2 J_\mu(y') - y'^3 J_{\mu+1}(y') \right] \end{aligned}$$

Appendix B

An orthonormal set is defined through a scalar product in the vector space of the solutions of some equation of motion. In the case of Klein-Gordon field this scalar product is Hermitian but not positive definite. Let be $f(x)$ and $g(x)$ two elements of F , where F is the vector space of the solutions of the Klein-Gordon equation with the scalar product defined by

$$(f, g) = -i \int_\Sigma \sqrt{-g} d\Sigma^\mu [f(x)(\partial_\mu g^*(x)) - (\partial_\mu f(x))g^*(x)]$$

where $d\Sigma^\mu = \eta^\mu d\Sigma$ with η^μ a future directed unit vector orthogonal to the space-like hypersurface Σ and $d\Sigma$ is the volume element in Σ . An orthonormal set (u_k, u_k^*) is said to be complete if every solution $f(x)$ of F can be written as

$$f(x) = \sum_k a_k u_k(x) + b_k u_k^*(x)$$

where the coefficients a_k and b_k are given by

$$a_k = (u_k, f)$$

and

$$b_k = -(u_k^*, f).$$

Let G be a subset of F . If (v_j, v_j^*) and (u_i, u_i^*) are two orthonormal sets such that the expand every element of G , then they are called equivalents. In this case

$$v_j(x) = \sum_i \alpha_{ji} u_i(x) + \beta_{ji} u_i^*(x)$$

and

$$u_i(x) = \sum_j \alpha_{ji}^* v_j(x) - \beta_{ji} v_j^*(x).$$

They are said to be complete only if $F = G$. The quantum field $\varphi(x)$ can be expanded using either of the two complete sets (u_i, u_i^*) or (v_j, v_j^*) that would lead to two different vacua $|0\rangle$ and $|0'\rangle$ respectively. When $\sum_{ij} |\beta_{ij}|^2$ converges, the representations are said to be unitarily equivalent. If it diverges they are non-unitarily equivalents and they are not related to any unitary operator in the Fock space [63, 64].

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